Solar Particle Events and Self-Organized Criticality: Are Deterministic Predictions of Events Possible?

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***Abstract*—Evidence is presented that solar particle events are a self-organized critical phenomenon. Using daily and monthly fluences of solar protons measured by the IMP-8 and GOES satellite instrumentation over a 28-year period, long-term correlation of events, fractal characteristics and power function behavior for the density functions of fluence magnitudes and waiting times are demonstrated. The implications are that it is not possible to predict the time of occurrence and magnitude of solar particle events within narrow limits.**

***Index Terms*—Fractal, rescaled range, self-organized criticality, solar particle event.**

I. Introduction

ORGANIZATIONS such as NASA, ESA, and others have put substantial resources into space missions designed to study properties of the sun. A few examples of such missions are the Solar and Heliospheric Observatory (SOHO), the Advanced Composition Explorer (ACE) and the Solar Dynamics Observatory (SDO). Many properties related to the occurrence of solar particle events are being studied such as the sun's magnetic topology and characteristics of radiation emitted during events in the hopes that some reliable predictor of events can be found. Despite this significant international effort, solar particle events can occur suddenly and without obvious warning. This is a serious difficulty for new space initiatives that plan to send manned spacecraft to the moon, Mars or interplanetary space. It can also be problematic for instrumentation as well as electronic and photonic subsystems. Thus, there is strong motivation to develop predictive methods for solar particle events. It is hoped that the apparent statistical character can be overcome and predictability achieved if precursor phenomena such as x-ray flares or magnetic topology signatures can be properly interpreted or if the underlying mechanisms are identified. This paper addresses the very basic question of whether deterministic predictions of solar particle events are possible (i.e., whether it is possible to predict their time of occurrence and magnitude).

We apply several statistical methods of analysis to satellite measurements of solar proton event fluences. These include rescaled range $(\mathrm{R/S})$ analysis, a method used to study long-term correlation of events. Fractal properties of event sizes are then demonstrated, suggesting a "scale invariant" behavior. Finally integral distributions for fluence magnitudes and waiting times between events are shown to be power functions. These analyses reveal features that are the hallmarks of systems exhibiting 'self-organized criticality' [1]. According to the theory of self-organized criticality (SOC), a slow continuous build-up of energy in a large interactive system causes the system to evolve to a critical state. A minor, localized disturbance can then start an energy-releasing chain reaction. Chain reactions of all sizes are an integral part of the dynamics, leading to a "scale invariant" property for event sizes. The scale invariance is manifested as power function distributions for the density functions of event magnitudes and waiting times. As a result of this basic nature it is generally assumed in the literature that accurate predictions of the magnitude and time of occurrence of such events are not possible.

Applications for the theory of SOC have been found for natural phenomena such as earthquakes, avalanches and rainfall. It was originally thought that a sandpile would be a simple system that exhibited SOC [1], [2]. However, it turns out that experiments do not support this and have led to modified models. The sandpile, however, remains useful as a conceptual aid. If sand is dropped one grain at a time to form a pile, the pile soon becomes large enough that grains may slide down it, thus releasing energy. Eventually the slope of the pile is steep enough that the amount of sand added is balanced, on average, by the amount that slides down the pile. At this point the system is in the critical state. As single grains of sand are added to the system in a critical state, a broad range of consequences are possible. Nothing may happen or an avalanche of any size up to a "catastrophic" one may occur. The complex dynamics of this interactive system prevent accurate predictions of when an avalanche will occur or how large it will be. In this paper, we demonstrate that energy release due to solar particle events is consistent with the dynamics of an SOC system. We also discuss the implications of this finding.

II. Solar Proton Data

Data from the Interplanetary Monitoring Platform-8 (IMP-8) and geostationary operational environmental satellites (GOES) have been combined to allow the best features of each data set to be taken advantage of [3]. The time period of the data used here is from November 1973 through October 2001, which spans 28 years. In this work, we have integrated measured fluxes over time to obtain daily and monthly solar proton fluences. Note that this differs from the usual analysis in which *event* fluences are used. The use of daily and monthly fluences is consistent with the statistical approaches used in this work and also avoids the problem of defining events, which can have a complex time dependence. An example of the data processed in this manner

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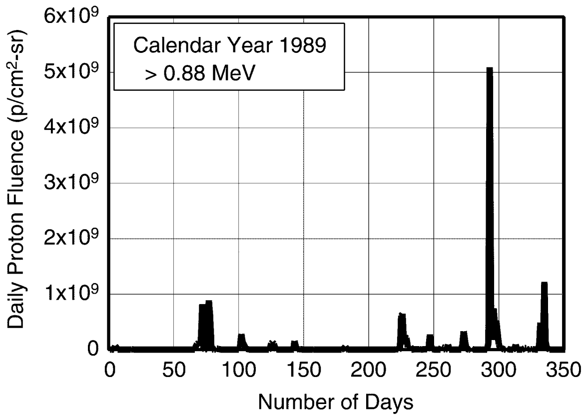

Fig. 1. Number and magnitude of solar proton daily fluences >0.88 MeV in 1989.

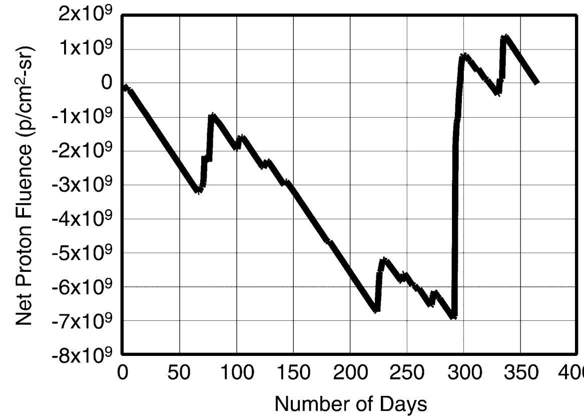

Fig. 2. Data in Fig. 1 as a cumulative deviation plot for $\mathrm{R/S}$ analysis.

is given in Fig. 1, which shows the >0.88 MeV daily proton fluences for calendar year 1989. A detailed description of the solar proton data-base is given in [3].

III. Results and Discussion

A. Rescaled Range Analysis

$\mathrm{R/S}$ analysis, originated by Hurst [4], turns out to be closely related to the theory of SOC. The original goal of Hurst was to provide a basis for estimating the optimum size of water storage reservoirs. An optimum size was taken as a reservoir that never ran dry or overflowed. The analysis was based on a history of floods and droughts in the region of interest over a period of many years. For a period of years beginning at time t the cumulative input to the reservoir is

$$Y_{t+\tau} = \sum_{i=t}^{t+\tau} X_i \tag{1}$$

where the X_i are the observed inputs for a given time interval (i.e., the daily or monthly input). The cumulative deviation for the total observation period of τ years is then

$$\Delta Y_{t+\tau} = \sum_{i=t}^{t+\tau} (X_i - \overline{Y_{t+\tau}}) \tag{2}$$

where $\overline{Y_{t+\tau}}$ is the mean value of the stochastic quantity X_i. Thus, the cumulative deviation represents the difference between the actual cumulative input to the reservoir at a given time and a cumulative calculation based upon the average inflow over the total time period of interest. This analysis permits identification of the maximum cumulative input and the value of the minimum cumulative store thereby enabling identification of the optimum size of the reservoir. The difference between the maximum and minimum values is customarily defined as the range.

In order to compare results for different rivers Hurst rescaled the range by dividing it by the standard deviation of the inputs over the period of the record τ. It turns out that this rescaled range is given by

$$R/S = a\tau^H \tag{3}$$

where a and H are constants [5]. The latter constant is called the Hurst coefficient. It is known that if the inputs are completely random and uncorrelated the rescaled range should vary as the square root of the elapsed time (i.e., H would equal 0.5). Contrary to this expectation, Hurst found that the rescaled range varied as the 0.7 to 0.8 power of the elapsed time indicating that the events showed long-term correlation. He found that many other natural phenomena such as rainfall, temperatures, pressures, and sunspot numbers had power indices in the same range.

Next, we show that if rescaled range analysis is applied to solar proton fluences, the analysis yields an index comparable to those found by Hurst for other natural phenomena. Measurements of statistical processes such as solar proton fluences are often presented as shown in Fig. 1. Here, it is seen that a very large event beginning at day 292 and extending to day 295 contributed most of the proton fluence for 1989.

In Fig. 2 a plot analogous to that used by Hurst to describe flood and drought periods is shown for solar proton daily fluences in the year 1989. The quantity plotted on the ordinate, the cumulative deviation or net proton fluence, is the analog of the reservoir level in Hurst's analysis. A negative slope on this plot indicates a lack of solar proton events (a "solar proton drought"). When an event occurs there is a rapid increase in the net proton fluence, producing the jagged appearance of the plot. It can be deduced from Figs. 1 and 2 that there is likely a continuous energy build-up with time that is released in bursts. Furthermore, there is a large range of solar proton event sizes that are possible. The events appear to occur randomly in time and have apparently random magnitudes. These are characteristics of an SOC process.

Fig. 2 shows the minimum cumulative deviation from the average at day 291 is -6.92×10^9 and the maximum at day 336 is $+1.40 \times 10^9$. The total difference amounts to 8.32×10^9 p/(cm^2-sr). In $\mathrm{R/S}$ analysis this is conventionally referred to as the range and when divided by the standard deviation yields the rescaled range. To carry out a complete $\mathrm{R/S}$ analysis a number

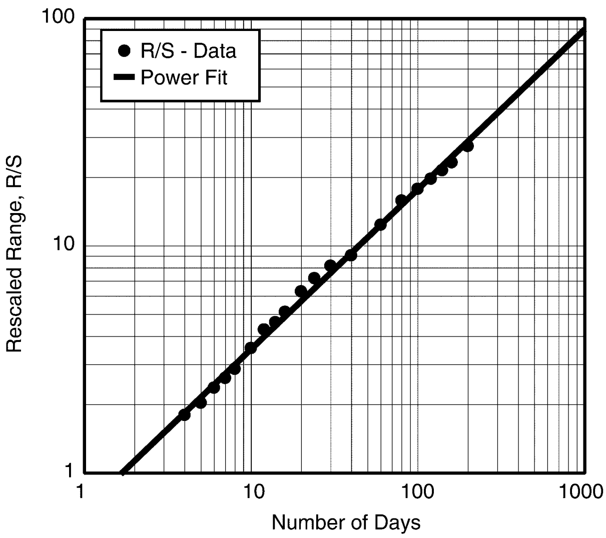

Fig. 3 $\mathrm{R/S}$ analysis of >0.88 MeV protons for 1989.

of samples covering different time periods in the total record are used to determine a series of rescaled range values. In the present case the sample sizes were varied from 4 to 200 days. When $\mathrm{R/S}$ values are amenable to this analysis, they yield a straight line when plotted as a function of the period on a log-log scale. As seen in Fig. 3 the solar proton data are well described by rescaled range analysis. This can next be used to determine the power index, H, in (3). Fitting the data results in a power index of 0.70, which is typical of those for other natural phenomena and indicates long-term correlation between solar particle events. This can be interpreted as a consequence of the fact that the amount of energy stored in the system (i.e., the sun's corona), is dependent on the system's past history. This behavior is consistent with a system that is in an SOC state.

B. Fractal Behavior

An interesting characteristic of the data in Fig. 2, which is a significant feature resulting from a system in an SOC state, is that when it is viewed on a different time scale, the character of the appearance does not change. This behavior is shown in Fig. 4, which plots the net proton fluence as a function of monthly fluences as opposed to Fig. 2, which is for daily fluences. Fig. 4 covers the entire 28-year range of data. If the units on the axes were not visible it would not be possible to determine the length of the record. For this reason processes of this type have been described in the literature by terms such as "scale invariant", "self-similar" and "fractal" [6]–[8]. This type of scale invariance is an indication that power function behavior for the density function distributions for fluence magnitudes and waiting times are possible, which are important characteristics of an SOC system. In fact, it has been suggested that a fractal can be thought of as a snapshot of an SOC process [2].

In order to illustrate the differences in this analysis from a solar process that does not display SOC behavior, $\mathrm{R/S}$ analysis has been applied to the sunspot record over the same 28 years covered in Fig. 4. Results are shown in Fig. 5. The smoothly varying curve is strikingly different than what was found for solar proton fluences. However, it is interesting to note that the periodicity shown in Fig. 4, although highly statistical, generally correlates with the periodicity shown in Fig. 5.

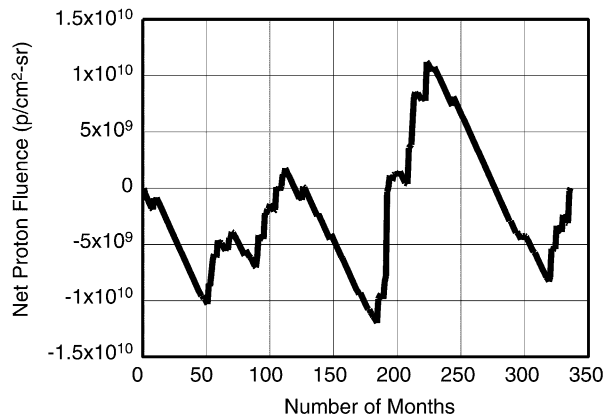

Fig. 4. Cumulative deviation plot for >0.88 MeV protons for the time period 1973 to 2001.

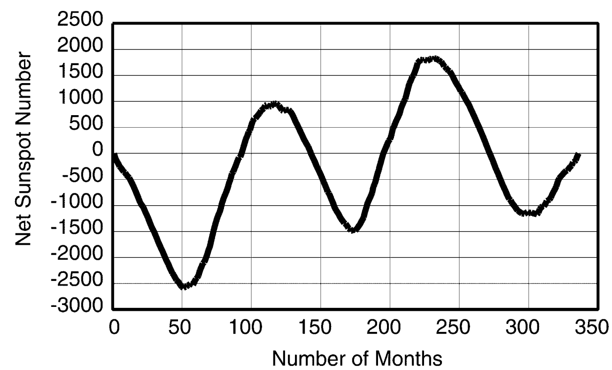

Fig. 5. Cumulative deviation plot of the monthly sunspot record from 1973 to 2001.

C. Power Function Distributions

Long-term correlation of events and fractal behavior such as those results discussed in sections A and B above are often seen for processes identified as manifesting SOC. An additional characteristic of such processes is that their number density distribution is a power function [5]–[7]. To examine this behavior, an integral distribution of monthly fluences for the 28-year period has been constructed and is shown in Fig. 6. The ordinate represents the number of occurrences when the monthly fluence exceeds that shown on the abscissa. It is seen that this distribution is a straight line on a semi-log plot that spans about 4 orders of magnitude. The very smallest fluences shown are close to the detector background levels and it is believed that is why the distribution starts to fall below the straight line at fluences approaching 10^5 p/(cm^2-sr). The data in Fig. 6 were fit to a function of form $N(\Phi) = b - c\ln(\Phi)$, where Φ is the proton fluence, and b and c are constants. The number density function is then

$$\frac{dN}{d\Phi} = \frac{-29.4}{\Phi} \tag{4}$$

In general, a power function result for the number density is regarded as a necessary condition for SOC. In this particular case the density function turns out to be exactly proportional to

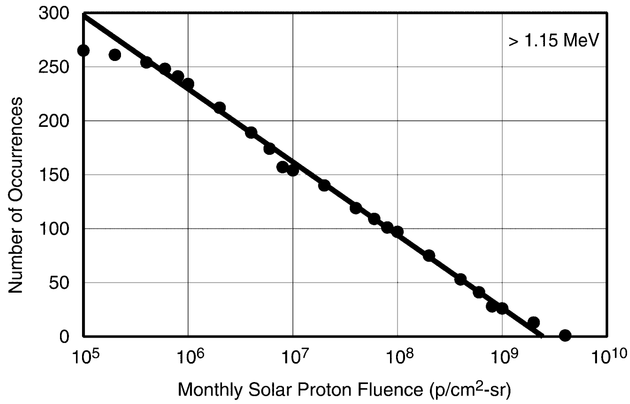

Fig. 6. Integral distribution of monthly solar proton fluences >1.15 MeV, from 1973 to 2001.

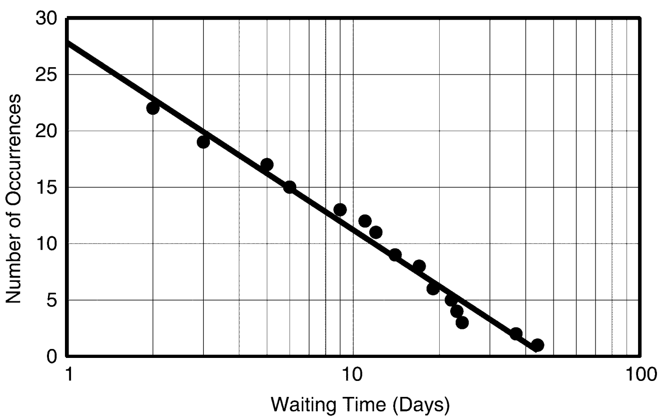

Fig. 7. Integral distribution of waiting times for solar proton events >0.88 MeV in calendar year 1989.

the reciprocal of the fluence. Thus, the solar event data can be represented by a power function of a type commonly referred to as $1/f$ [1]. It is well known that $1/f$ noise, or flicker noise, suggests that the dynamics of a system are strongly influenced by past events. This gives further support to the results shown in Section A. It has been suggested that flicker noise results when a system in the critical state produces events with broad ranges of size and duration [2].

Further evidence along this line of reasoning is shown by the integral distribution of waiting times between events. As an example, Fig. 7 shows this distribution for calendar year 1989. A procedure analogous to that used to derive (4) was used to obtain the density function for waiting times. The result is

$$\frac{dN}{dT} = \frac{-7.2}{T} \tag{5}$$

where T is the time between events. As was the case for the monthly fluence magnitudes, it is found that the density function for waiting times follows a $1/f$ power function exactly. The probability density functions for both the fluence magnitudes and the waiting times are power functions—the hallmark of SOC phenomena. Furthermore, the fact that both power functions follow $1/f$ behavior exactly makes an especially compelling argument for an SOC system.

IV. Summary and Conclusions

We have applied several statistical techniques to a comprehensive satellite data base of solar proton flux measurements. Research involving SOC is still a developing field [9]–[16], and there is much yet to be learned about the sun's dynamics. However, our results strongly indicate that solar particle events are an SOC phenomenon. The general physical behavior of an SOC system is that of a non-equilibrium system driven by a slow continuous energy input that eventually is released in sudden bursts with no typical size (i.e., the energy release is scale invariant). This corresponds to the power function distribution shown in (4). There are a variety of other processes that occur in nature that have been identified with these features such as earthquakes, avalanches and rainfall. For example, the power function distribution of earthquakes, known as the Gutenberg–Richter Law, is analogous to (4) for solar particle fluences.

Lu and Hamilton have proposed that the solar coronal magnetic field is in an SOC state and therefore a localized change in the sun's magnetic topology can initiate solar x-ray flares that have a peak intensity distribution given by a power function [15], [16]. Although we have not drawn any conclusions about the mechanisms involved, we note that our observations about solar particle events may be explained by reasoning similar to that of Lu and Hamilton. To our knowledge this is the first time that solar particle events have been identified as an SOC phenomenon. Thus, these results strongly suggest that it is not possible to predict that a solar particle event of a given size will occur at a given time. In this respect, solar particle events are analogous to earthquakes where a similar conclusion has been reached [17]. This is consistent with our previous observation of the mathematical similarities between earthquake and solar particle event statistics [18].

The lack of predictability of solar particle events has consequences with regard to manned missions to the moon or Mars. It underscores the importance of establishing a measurement system in the inner heliosphere for early detection and warning of events. In addition it indicates that solar particle event modeling for future missions will continue to draw on techniques from the field of probability theory. In this respect, the connections made here to the theory of SOC should considerably broaden the range of modeling techniques that can be focused on solar particle event data. For example, the findings that the magnitudes and waiting times are described by $1/f$ power laws makes the connection to the extensive literature on noise phenomena.

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